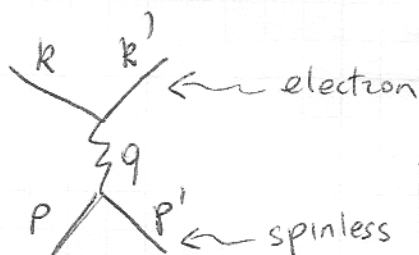


PHYSICS 225A • HOMEWORK 7

1

① H&M 6.8



$$M = -e^2 u(k') \delta^h u(k) \frac{1}{q^2} (p+p')_\mu$$

using Feynman rules

Then $|\overline{M}|^2 = \frac{e^2}{q^4} L_e^{\mu\nu} L_{\mu\nu}^{\text{spinless}}$

$$L_{\mu\nu}^{\text{spinless}} = (p+p')_\mu (p+p')_\nu$$

$$L_e^{\mu\nu} = 2 [k'^\mu k^\nu + k'^\nu k^\mu - (kk') g^{\mu\nu}]$$

H.M., 6.25, m=0

To save some work, note $q_\mu L^{\mu\nu} = 0$

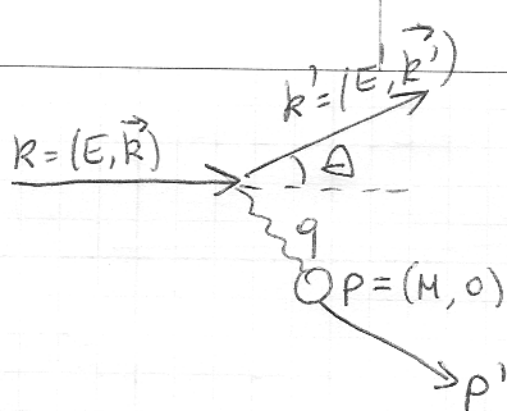
$$\text{Then } p'_\mu L_e^{\mu\nu} = (p_\mu + k_\mu - k'_\mu) L_e^{\mu\nu} = (p_\mu + q_\mu) L_e^{\mu\nu} = p_\mu L_e^{\mu\nu}$$

Then

$$|\overline{M}|^2 = \frac{e^2}{q^4} 2 [k'^\mu k^\nu + k'^\nu k^\mu - (kk') g^{\mu\nu}] [(2p_\mu)(2p_\nu)]$$

$$|\overline{M}|^2 = \frac{8e^2}{q^4} [2(k'p)(kp) - M^2(kk')]$$





$$k'p = ME'$$

$$kp = ME$$

$$kR' = \frac{2}{3} EE' \sin^2 \frac{\theta}{2} \quad (\text{H.M. 6.44})$$

$$|M|^2 = \frac{8e^2}{q^4} \left[2M^2 EE' - 2M^2 EE' \sin^2 \frac{\theta}{2} \right] = \frac{8e^4}{q^4} 2M^2 EE' \cos^2 \frac{\theta}{2}$$

Compare this result with Helzen and Martin, equation 6.43
Carrying it through just like what is done in H&M between 6.43 and 6.50 gives

$$\frac{d\sigma}{d\Omega} \Big|_{\text{LAB}} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \frac{E'}{E} \cos^2 \frac{\theta}{2}$$

② H&M 8.6

$$p + q = p' \quad (p + q)^2 = p'^2$$

$$p^2 + 2pq + q^2 = p'^2$$

$$M^2 + 2pq + q^2 = M^2 \Rightarrow \boxed{pq = -\frac{1}{2}q^2}$$

③ H&M 8.11

From equation 8.29 $W^2 = M^2 + 2M\nu + q^2$

$$2M\nu + q^2 = W^2 - M^2$$

~~But~~ $W^2 \geq M^2$ - So one limiting case $2M\nu + q^2 = 0$

$$\text{Or } x = -\frac{q^2}{2M\nu} = 1$$

The other limiting case is when $q^2=0$ (note that $q^2 < 0$)
 This gives $x=0 \Rightarrow \boxed{x \in [0, 1]}$

Now $y = \frac{pq}{pk}$

Work in frame where proton is at rest

HM, 6.45 $pq = -\frac{1}{2}q^2$

HM, 6.44 $q^2 = -4EE'\sin^2\frac{\theta}{2}$

$\Rightarrow pq = 2EE'\sin^2\frac{\theta}{2}$

$pk = EM \Rightarrow y = \frac{2E'\sin^2\frac{\theta}{2}}{M}$ Minimum $y=0$ at $\theta=0$

Also, HM 6.45 $pq = \nu M$ and $\nu = E - E'$

So $y = \frac{(E-E')M}{EM} = 1 - \frac{E'}{E}$ Maximum $E'=0 \Rightarrow y=1$

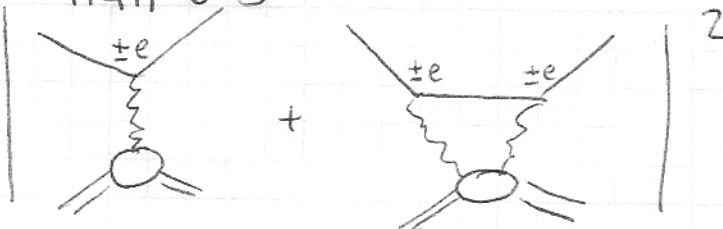
$\Rightarrow \boxed{y \in [0, 1]}$

④ H&M 8.12

$\nu = E - E'$ is shown in H&M, page 131

$y = \frac{E-E'}{E}$ was shown in the previous problem

⑤ H&M 8.13



graphical representation of the $|M|^2$

The sign is different for electron and positrons - We get $|M|^2 \sim Ae^2 \pm Be^3 + Ce^4$ (different for e^+ and e^-)

⑥ H&M 9.9

$$\frac{1}{x} F_2^{en}(x) = \frac{4}{9} d(x) + \frac{1}{9} u(x) + \frac{1}{9} s(x)$$

These are proton distribution functions, using SU(2) symmetry

$$\frac{1}{x} F_2^{ep}(x) = \frac{4}{9} u(x) + \frac{1}{9} d(x) + \frac{1}{9} s(x)$$

$$\frac{F_2^{en}(x)}{F_2^{ep}(x)} = \frac{4d(x) + u(x) + s(x)}{d(x) + 4u(x) + s(x)}$$

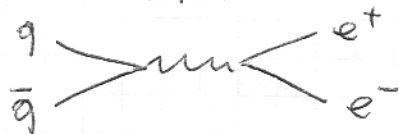
The limiting cases are $d(x) = s(x) = 0 \Rightarrow \frac{F_2^{en}(x)}{F_2^{ep}(x)} = \frac{1}{4}$

$u(x) = s(x) = 0 \Rightarrow \frac{F_2^{en}(x)}{F_2^{ep}(x)} = 4$

⑦

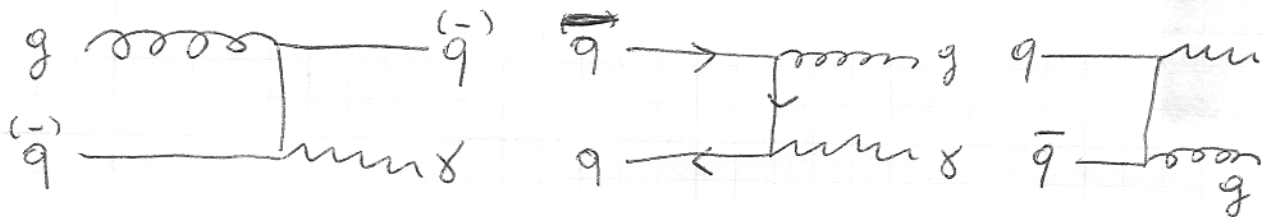
$$pp \rightarrow e^+ e^- X$$

At parton level this must be $q\bar{q} \rightarrow e^+ e^-$



Since $\bar{q}$ in the proton can only come from the sea, this cross section gives info on the sea distribution

$$p\bar{p} \rightarrow \gamma + X \quad g\bar{q}' \rightarrow \gamma q' \quad q\bar{q} \rightarrow g\gamma$$



$$\textcircled{8} \quad \frac{1}{x} F^{ep}(x) = \frac{4}{9} u(x) + \frac{1}{9} d(x) + \frac{1}{9} s(x)$$

$$\frac{1}{x} F^{en}(x) = \frac{4}{9} d(x) + \frac{1}{9} u(x) + \frac{1}{9} s(x)$$

(same as problem 6)

Then

$$\frac{1}{x} [F^{ep}(x) - F^{en}(x)] = \frac{3}{9} u(x) - \frac{3}{9} d(x) = \frac{1}{3} [u(x) - d(x)]$$

$$\begin{aligned} \int \frac{dx}{x} [F^{ep}(x) - F^{en}(x)] &= \frac{1}{3} \int dx u(x) - \frac{1}{3} \int dx d(x) \\ &= \frac{1}{3} \cdot 2 - \frac{1}{3} \cdot 1 = \frac{1}{3} \checkmark \end{aligned}$$